ASSIGNMENT #10

Please do not write your answers on a copy of this assignment, use blank paper. As with all assignments, there will conceptual and computational questions. For computational problems you may check your work using any tool you wish; however **you must clearly explain each step that you make in your computation.**

For this assignment I encourage you to work with others; however, you are expected to **submit your own work in your own words**. In addition to the true and false section being graded, I will grade one other problem; this will account for 10 points out of 25. The other 15 will be based on completion. **If you would like feedback on a particular problem, please indicate it somehow.** You must make an honest attempt on each problem for full points on the completion aspect of your grade.

(1) Find the eigenvalues of the following matrices. After you find their eigenvalues, describe each eigenvalue's eigenspace using set-builder notation.

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 3 \\ -4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -4 & 0 & 2 \\ 0 & -3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(2) Find a basis for the eigenspace corresponding the each listed eigenvalue below.

(a) $\begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}$; $\lambda = 1, 5$

(b) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 3 & 3 \\ 6 & 6 & 2 \end{bmatrix}$; $\lambda = -4$

(3) Is $\lambda = 4$ an eigenvalue of $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$? If it is find one corresponding eigenvector.

(4) Is $\begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 3 & 7 & 9 \\ -4 & -5 & 1 \\ 2 & 4 & 4 \end{bmatrix}$? If so, find its eigenvalue.

(5) Find an eigenvector for the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

(6) Find the eigenvalues of the 2×2 matrix that roates points by 45 degrees about the origin. For each eigenvalue you find, find a basis for its eigenspace.

(7) For each of the following matrices, determine if the matrix is diagonalizable, and if so find P and D such that $A = PDP^{-1}$, where D is a diagonal matrix.

(a) $\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

(8) Answer the following true and false questions. No justification is required.

(a) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A

(b) The eigenvalues of a matrix are the entries of its diagonal.

(c) A square matrix A is invertible if and only if 0 is not an eigenvalue of A.

(d) A square matrix A and it's transpose have the same eigenvalues.

(e) A square matrix A of size n can have more than n eigenvalues.

(f) If two matrices have the same eigenvalues, then they are similar.

(g) A square matrix A of size n is diagonalizable if and only if it has n linearly independent eigenvectors.

(h) $A = PBP^{-1}$ and $A = QBQ^{-1}$, then $P = Q$.

(i) Let A be an $n \times n$ matrix. If the sum of the dimensions of the eigenspaces for A is equal to n, then A is diagonalizable.